Lecture #27: More Scheme Programming

Recursion and Iteration

- We've mentioned before that Scheme uses recursion where most other languages (such as Python) use special iterative constructs.
- This puts a special burden on Scheme interpreters to handle iterative recursions, known as *tail recursions* well.
- From the reference manual:

  > "Implementations of Scheme must be *properly tail-recursive.* Procedure calls that occur in certain syntactic contexts called *tail contexts* are tail calls. A Scheme implementation is properly tail-recursive if it supports an *unbounded number of [simultaneously] active tail calls.*

- First, let's define what that means.

Tail Contexts

- Basically, an expression is in a *tail context* if it is evaluated last in a function body and provides the value of a call to that function.
- A function is *tail-recursive* if all function calls in its body that can result in a recursive call on that same function are in tail contexts.
- In effect, Scheme turns recursive calls of such functions into iterations by *replacing* those calls with one of the function's tail-context expressions instead of simply returning.
- This decreases the memory devoted to keeping track of which functions are running and who called them to a constant.

Tail Contexts in Scheme

- Tail contexts are defined inductively (or recursively). The "bases" are

  (lambda (*ARGUMENTS*) $EXPR_1$ $EXPR_2$ ... $EXPR_n$) ; Tail contexts in Blue
  (define (*NAME ARGMENTS*) $EXPR_1$ $EXPR_2$ ... $EXPR_n$)

  ('*EXPR*' means "Scheme expression")

- If an expression is in a tail context, then certain parts of it become tail contexts all by themselves:

  (if *EXPR* *THEN-EXPR* *ELSE-EXPR*)

  (cond (*COND-EXPR*$_1$ $EXPR_{11}$ $EXPR_{12}$ ... $EXPR_{1n}$)
  (*COND-EXPR*$_2$ $EXPR_{21}$ $EXPR_{22}$ ... $EXPR_{2n}$)
  ...)

  (and $EXPR_1$ ... $EXPR_n$)
  (or $EXPR_1$ ... $EXPR_n$)

  (begin $EXPR_1$ ... $EXPR_n$)

Prime Numbers

```
(define (prime? x)
  "True iff X is prime."



)
```


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  (cond ((< x 2) #f)
        ((= x 2) #t)
        (#t ?))
)
```

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(define (prime? x)
  "True iff X is prime."

  (define (no-factor? k lim)
    "LIM has no divisors >= K and < LIM."
    )

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        ((= x 2) #t)
        (#t (no-factor? 2
                        (floor (+ (sqrt x) 2)))))
)
```


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```
(define (prime? x)
  "True iff X is prime."

  (define (no-factor? k lim)
    "LIM has no divisors >= K and < LIM."
    (cond ((>= k lim) #t)
          ((= (remainder x k) 0) #f)
          (#t ?)))

  (cond ((< x 2) #f)
        ((= x 2) #t)
        (#t (no-factor? 2
                        (floor (+ (sqrt x) 2)))))
)
```


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```
(define (prime? x)
  "True iff X is prime."

  (define (no-factor? k lim)
    "LIM has no divisors >= K and < LIM."
    (cond ((>= k lim) #t)
          ((= (remainder x k) 0) #f)
          (#t (no-factor? (+ k 1) lim))))

  (cond ((< x 2) #f)
        ((= x 2) #t)
        (#t (no-factor? 2
                        (floor (+ (sqrt x) 2)))))
)
```


Tail-Recursive Length?

- On several occasions, we've computed the length of a linked list like this:

```
;; The length of list L
(define (length L)
    (if (eqv? L '())       ; Alternative: (null? L)
        0
        (+ 1 (length (cdr L)))))
```

  but this is not tail recursive. How do we make it so?

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  but this is not tail recursive. How do we make it so?
- Try a helper method:

```
;; The length of list L
(define (length L)
    (define (length+ ?)

        )
    (length+ ?))
```

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```
;; The length of list L
(define (length L)
    (define (length+ n R)
        "The length of R plus N."
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    (length+ 0 L))
```


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but this is not tail recursive. How do we make it so?

- Try a helper method:

```
;; The length of list L
(define (length L)
    (define (length+ n R)
        "The length of R plus N."
        (if (null? R) n
            (length+ (+ n 1) (cdr R))))
    (length+ 0 L))
```


Standard List Searches: assoc, etc.

- The functions assq, assv, and assoc classically serve the purpose of Python dictionaries.
- An *association list* is a list of key/value pairs. The Python dictionary {1 : 5, 3 : 6, 0 : 2} might be represented

  ((1 . 5) (3 . 6) (0 . 2))

- The ass$x$ functions access this list, returning the pair whose car matches a key argument.
- The difference between the methods is that
  - assq compares using eq? (Python **is**).
  - assv uses eqv? (which is like Python == on numbers and like **is** otherwise).
  - assoc uses equal? (does "deep" comparison of lists).

```
;; The first item in L whose car is eqv? to key, or #f if none.
(define (assv key L)
)
```


Assv Solution

```
;; The first item in L whose car is eqv? to key, or #f if none.
(define (assv key L)
    (cond ((null? L)          #f)
          ((eqv? key (caar L)) (car L))
          (else               (assv key (cdr L))))
)
```

- This is a tail-recursive function.
- Why caar ((car (car ...)))?
  - L has the form ((key1 . val1) (key2 . val2) ...).
  - So the car of L is (key1 . val1), and its key is therefore (car (car L)) (or caar for short).

A classic: reduce

```
;; Assumes f is a two-argument function and L is a list.
;; If L is (x1 x2...xn),  the result of applying f n-1 times
;; to give (f (f (... (f x1 x2) x3) x4) ...).
;; If L is empty, returns f with no arguments.
;; [Simply Scheme version.]
;; >>> (reduce + '(1 2 3 4)) ===> 10
;; >>> (reduce + '()) ===> 0
(define (reduce f L)


)
```


Reduce Solution (1)

```
;; Assumes f is a two-argument function and L is a list.
;; If L is (x1 x2...xn),  the result of applying f n-1 times
;; to give (f (f (... (f x1 x2) x3) x4) ...).
;; If L is empty, returns f with no arguments.
(define (reduce f L)
    (cond ((null? L)
           (f))       ; Odd case with no items
          ((null? (cdr L))
           (car L))   ; One item
          (else (reduce f (cons (f (car L) (cadr L))
                                (cddr L))))))
; E.g.:
;    (reduce + '(2 3 4))
;    -calls->  (reduce + (5 4))
;    -calls->  (reduce + (9))
;    -yields-> 9
```

Reduce Solution (2)

```
;; Assumes f is a two-argument function and L is a list.
;; If L is (x1 x2...xn),  the result of applying f n-1 times
;; to give (f (f (... (f x1 x2) x3) x4) ...).
;; If L is empty, returns f with no arguments.
(define (reduce f L)
    (define (reduce-tail accum R)
       (cond ((null? R) accum)
             (else (reduce-tail (f accum (car R)) (cdr R)))))

    (if (null? L) (f)   ;; Special case
        (reduce-tail (car L) (cdr L))))
```


A Harder Case: Map

- We've seen map many times.
- An obvious Scheme version:

```
;; Assuming f is a one-argument function and L a list, the list of
;; results of applying f to each element of L
(define (map f L)
    (if (null? L) ()
        (cons (f (car L) (map f (cdr L))))))
```

- Is this tail-recursive?

Making map tail recursive

- Need to pass along the partial results and add to them.
- Problem: cons adds to the *front* of a list, so we end up with a reverse of what we want.

```
(define (map f L)
     ;; The result of prepending the reverse of (map rest) to
     ;; the list partial-result
     (define (map+ partial-result rest)
         (if (null? rest) partial-result
             (map+ (cons (f (car rest)) partial-result)
                   (cdr rest))))
     (reverse (map+ () L)))
```

- What about reverse?

And Finally, Reverse

- Actually, we can use the very problem that cons creates to solve it!
- That is, consing items from a list from left to right results in a reversed list:

```
(define (reverse L)
    (define (reverse+ partial-result rest)
        (if (null? rest) partial-result
            (reverse+ (cons (car rest) partial-result)
                      (cdr rest))))
    (reverse+ () L))
```


Another Example

- Consider the problem of shuffling together two lists, L1 and L2. The result consists of the first item of L1, then the first of L2, then the second of L1, etc., until one or the other list has no more values.
- Obvious recursive solution:

```
(define (shuffle1 L1 L2)
  "The list consisting of the first element of L1, then"
  "the first of L2, then the second of L1, etc., until"
  "the elements of one or the other list is exhausted."
  (if (null? L1) '()
      ?))
```


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  (if (null? L1) '()
      (cons (car L1) (shuffle1 L2 (cdr L1)))))
```

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  "the elements of one or the other list is exhausted."
  (if (null? L1) '()
      (cons (car L1) (shuffle1 L2 (cdr L1)))))
```

- Not tail recursive. Again, we can use a helper method:

```
(define (shuffle L1 L2)
  (define (shuffle+ reversed-result L1 L2)
    (if ?))
  (shuffle+ '() L1 L2))
```


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(define (shuffle L1 L2)
  (define (shuffle+ reversed-result L1 L2)
    (if (null? L1) (reverse reversed-result)
        ?))
  (shuffle+ '() L1 L2))
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- Not tail recursive. Again, we can use a helper method:

```
(define (shuffle L1 L2)
  (define (shuffle+ reversed-result L1 L2)
    (if (null? L1) (reverse reversed-result)
        (shuffle+ (cons (car L1) reversed-result) L2 (cdr L1))))
  (shuffle+ '() L1 L2))
```