Lecture #20: Search and Sets Revisited

Container Objects and Searching

- Lists, linked lists, trees, and dictionaries are various objects whose principle purpose is to *contain values* and present them in various ways.
- We've principally considered operations that involve retrieving all values and doing something with them.
- But a central activity of many programs and algorithms is *finding* a value that meets certain criteria *in* one of these containers.
- Several Python data structures provide methods for finding:

```
x in aList     # Is x in aList?
x in aDict     # Is x a key in aDict?
aDict[x]       # What is V if aDict contains the entry (x, V)?
"61A" in text  # Does substring '61A' appear in string text?
```


Sets

- Current versions of Python also have *sets*, which are intended to behave like mathematical sets.
- Examples:

```
A = { 1, 3, 2 }     # Definition by extension
B = set([1, 3, 5])  # Contents can come from an iterable
set()               # The empty set
{}                  # The empty dictionary (sorry)
{ x for x in L if x % 2 == 1 }
                    # Set generator: odd members of L
                    # Like {x|x ∈ L and x is odd }
A | B  == { 1, 2, 3, 5 } == A.union(B) # A ∪ B
A & B == { 1, 3 } == A.intersection(B) # A ∩ B
A - B == { 2 } == A.difference(B) == { x for x in A if x not in B }
A < (A | B) == True # A ⊂ A ∪ B
3 in A == True      # 3 ∈ A
len(A) == 3         # |A| or size of A
```


Sets are Iterables

- Like other container types, one can iterate over sets.
- Python sets are *unordered:* ordering of iterator results is undefined.

```
>>> for x in { 5000, 3000, 100 }: print(x, end=" ")
3000 5000 100
>>> list( { 5000, 3000, 100 } )
[3000, 5000, 100]
```


Example

How can I test whether a list contains duplicates?

```
def hasDuplicates(L):
    """Return true iff list L contains duplicated values."""
```


Implementing Sets: Unordered Lists

- Clearly, lists also contain collections of values, so we could use them to implement sets.
- Must be careful to avoid duplicate elements (important when iterating).
- The algorithm for "member of" (`x in S`) is familiar:

```
def contains(S, x):
    """True iff list S (considered as a set) contains x."""
    for y in S:
        if x == y:
            return True
    return False
```

- If N is the length of `S`, what is the worst-case time bound?

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- If N is the length of `S`, what is the worst-case time bound? *Answer:* $\Theta(N)$

Implementing Sets: Insertion/Formation w/ Unordered List

What's the time required for this? Assume appending to a list takes $O(1)$ time (which is true on average).

```
def toSet(L):
    """Returns an unordered list containing all values in L without
    duplicates."""
    result = []
    for x in L:
        if not contains(result, x):
            result.append(x)
    return result
```


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Answer: $\Theta(N^2)$

Implementing Sets: Ordered Lists

- If we keep list sorted (say in ascending order), can use *binary search*:

```
def contains(S, x):
    """Returns true if X is in S, a list sorted in ascending order."""
    L, U = 0, len(S)-1
    while L <= U:
        M = (L + U) // 2
        if x == S[M]:
            return True
        elif x < S[M]:
            U = M - 1
        else:
            L = M + 1
    return False
```

- What's the execution time here (if N is `len(S)`)?

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```

- What's the execution time here (if N is `len(S)`)? *Answer:* $\Theta(\lg N)$

Implementing Sets: Insertion/Formation w/ Ordered List

What's the time required for this? Assume appending to a list takes $O(1)$ time (which is true on average).

```
def toSet(anIterable):
    """Returns an ordered list containing all values in ANITERABLE without
    duplicates."""
    result = []
    for x in anIterable:
        L, U = 0, len(result)-1
        while L <= U:
            M = (L + U) // 2
            if x == result[M]:
                break
            elif x < result[M]:
                U = M - 1
            else:
                L = M + 1
        if L > U:
          result.insert(L, x)
```

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Answer: $\Theta(N^2)$

Binary Search Trees

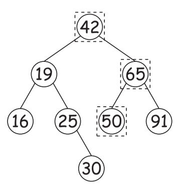
Binary Search Property:

- In a *binary tree*, each inner node has two children (called "left" and "right", typically), but trees are allowed to be *empty* (no label, no children).
- A *binary search tree* (BST) satisfies two other properties:
- All nodes in left subtree of a node have *smaller* keys.
- All nodes in right subtree of node have *larger* keys.
- This allows binary search, but in a tree.

Finding

- Example: Searching for 50 and 49 in a BST representing

  { 16, 19, 25, 30, 42, 50, 65, 91 }



```
def contains(S, x):
    """Returns true iff BST S contains x."""
    if S == BinTree.empty:
        return False
    if S.label == x:
        return True
    elif S.label < x:
        return contains(S.right, x)
    else:
        return contains(S.left, x)
```

- Dashed boxes show which node labels we look at.
- Number looked at proportional to height of tree.
- What is worst-case time (for a general tree with N nodes)?
- If tree is "bushy," what is worst-case time?

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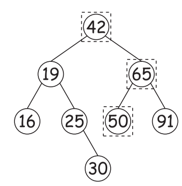

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- What is worst-case time (for a general tree with N nodes)? **Answer:** $\Theta(N)$
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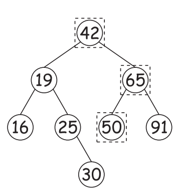

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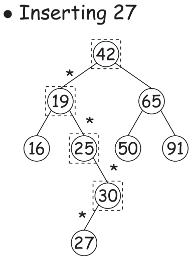
- Dashed boxes show which node labels we look at.
- Number looked at proportional to height of tree.
- What is worst-case time (for a general tree with N nodes)? **Answer:** $\Theta(N)$
- If tree is "bushy," what is worst-case time? **Answer:** $\Theta(\lg N)$

Inserting

- Inserting 27



```
def add(S, x):
    """Add X to binary search tree S destructively,
    if not already present, returning new tree."""
    if S == BinTree.empty:
        return BinTree(x)
    elif S.label < x:
        S.right = add(S.right, x)
    else:
        S.left = add(S.left, x)
    return S
```

- Starred edges are set (to themselves, unless initially null).
- Again, time proportional to height.

What Does Python Do?

- Binary trees are just a special case of this algorithm from Lecture #19:

```
def tree_find(T, disc):
    p = disc(T.label)
    if p == -1:
        return T.label
    elif T.is_leaf():
        return None
    else:
        return tree_find(T.children[p], disc)
```

  where the discrimination function (`disc`) returns either -1 (when the label is the target), 0 (for left child), or 1.

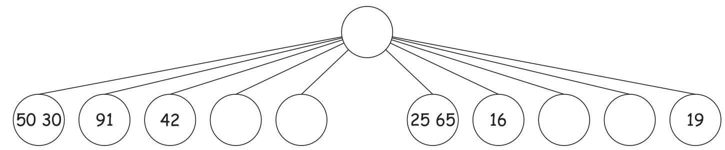
- In effect, for its sets (and dictionaries), Python uses another specialization of this same algorithm, where `disc` can return values in an arbitrary range, and the tree is always height 1.
- The discrimination function in this case is called a *hashing function*.

Hashing

- Example: the previous set of integers:

  `{ 16, 19, 25, 30, 42, 50, 65, 91 }`

  where the hashing function returns the value of the last digit.



- The tree labels on the leaves can be simple unordered lists of values, each sharing the same hashed value (their last digit in this case).
- As long as these lists stay small, look-up time is short. In fact, if there is a constant bound on list size, look-up time is $\Theta(1)$.
- When lists get too long, just increase the number of children at the root.

More Details

- To allow this to work, must define a hash function for your data.
- The Python way is to add a method `__hash__`, which is expected to return an integer such that the value returned for two objects that are considered equal (==) are equal.
- Python chooses the number of children (which are called *buckets*) of the top node depending on the current number of items in the set or dictionary represented.
- It then computes a discriminating value between between 0 and N, the number of children, by some process such as taking the value of `__hash__` modulo N.