Lecture #9: Sequences

- The term *sequence* refers generally to a data structure consisting of an *indexed collection of values.*
- That is, there is a first, second, third value (which CS types call #0, #1, #2, etc.
- A sequence may be *finite* (with a length) or *infinite.*
- As an object, it may be *mutable* (elements can change) or *immutable.*
- There are numerous alternative interfaces (i.e., sets of operations) for manipulating it.
- And, of course, numerous alternative implementations.
- Today: immutable, finite sequences, recursively defined.

A Recursive Definition

- A possible definition: A sequence consists of
  - An empty sequence, or
  - A first element and a sequence consisting of the elements of the sequence other than the first—the rest of the sequence or *tail.*
- The definition is clearly recursive ("a sequence consists of . . . a sequence . . . "), so let's call it an `rlist` for now.
- Suggests the following ADT interface:

```
empty_rlist = ...
def make_rlist(first, rest = empty_rlist):
    """A recursive list, r, such that first(r) is FIRST and
    rest(r) is REST,  which must be an rlist."""
def first(r):
    """The first item in R."""
def rest(r):
    """The tail of R."""
def isempty(r):
    """True iff R is the empty sequence"""
```


Implementation With Pairs

- An obvious implementation uses two-element tuples (pairs), such as those defined in lecture 8.
- The result is called a *linked list.*

```
empty_rlist = None
def make_rlist(first, rest = empty_rlist):
    return cons(first, rest)
def first(r):
    return left(r)
def rest(r):
    return right(r)
def isempty(r):
    return r is None
```


Implementation With Pairs (II)

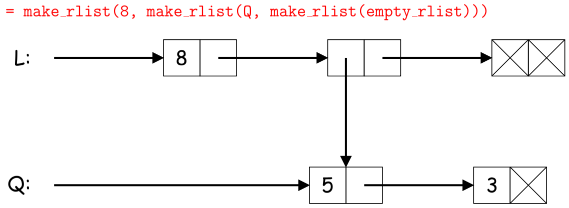
- This implementation is rather trivial. Basically, we've dnne nothing but give new names to the functions in the pair interface defined in lecture 8.
- In fact, we could have defined everything like this:

```
empty_rlist = None
make_rlist = cons
first = left
rest = right
def isempty(r):
    return r is None
```


Box-and-Pointer Diagrams for Linked Lists

- Diagrammatically, one gets structures like this:

```
# The sequence containing:  8; the sequence containing 5 and 3;
#    and the empty sequence
Q = make_rlist(5, make_rlist(3, empty_rlist))
L = make_rlist(8,
               make_rlist(Q, make_rlist(empty_rlist, empty_rlist)))
# or
# Q = make_rlist(5, make_rlist(3))
# L = make_rlist(8, make_rlist(Q, make_rlist(empty_rlist)))
```



From Recursive Structure to Recursive Algorithm

- The cases in the recursive definition of list often suggest a recursive approach to implementing functions on them.
- Example: length of an rlist:

```
def len_rlist(s):                # A sequence is:
    """The length of rlist 's'."""
    if isempty(s):               # Empty or...
        return 0
    else:
        return 1 + len_rlist(rest(s))
                                 # A first element and
                                 # the rest of the list
```

- **Q:** Why do we know the comment is accurate?
- **A:** Because we assume the comment is accurate! (For "smaller" arguments, that is).
- An example of reasoning by *structural induction. . .*
- . . . or *recursive thinking* about data structures.

Another Example: Selection

- Want to extract item #k from an rlist (number from 0).
- Recursively:

```
def getitem_rlist(s, i):
    """Return the element at index 'i' of recursive list 's'.
    >>> L = make_rlist(2, make_rlist(3, make_rlist (4)))
    >>> getitem_rlist(L, 1)
    3"""

    if ________:
        return ________
    else:
        return ________
```


getitem_rlist (II)

- Want to extract item #k from an rlist (number from 0).
- Recursively:

```
def getitem_rlist(s, i):
    "Return the element at index 'i' of recursive list 's'."

    if i == 0:
        return first(s)
    else:
        return getitem_rlist(rest(s), i-1)
```


Iterative Version of getitem_rlist

- Want to extract item #k from an rlist (number from 0).
- Recursively:

```
def getitem_rlist(s, i):
    "Return the element at index 'i' of recursive list 's'."

    if i == 0:
        return first(s)
    else:
        return getitem_rlist(rest(s), i-1)

def getitem_rlist(s, i):
    "Return the element at index 'i' of recursive list 's'."
    while ________________:

        ____, ____  = ________________

    return ________________
```


Iterative Version of getitem_rlist (II)

```
def getitem_rlist(s, i):
    "Return the element at index 'i' of recursive list 's'."

    if i == 0:
        return first(s)
    else:
        return getitem_rlist(rest(s), i-1)


def getitem_rlist(s, i):
    "Return the element at index 'i' of recursive list 's'."

    while i != 0:
        s, i = rest(s), i-1
    return first(s)
```


On to Higher Orders!

```
def map_rlist(f, s):
    """The rlist of values F(x) for each element x of rlist
       S (in the same order.)"""
    if ________:
        return ________
    else:
        return ________
```


Map implemented

```
def map_rlist(f, s):
    """The rlist of values F(x) for each element x of rlist
       S (in the same order.)"""
    if isempty(s):
        return empty_rlist
    else:
        return make_rlist(f(first(s)), map_rlist(f, rest(s)))
```

- So `map_rlist(lambda x:x**2, L)` produces a list of squares.
- [Python 3 produces a different kind of result from its `map` function; we'll get to it.]
- Iterative version not so easy here!

Filtering

- Map unconditionally applies its function argument to elements of a list. It is essentially a loop.
- The analog of applying an **if** statement to items in a list is called *filtering*:

```
def filter_rlist(cond, seq):
    """The rlist consisting of the subsequence of
    rlist 'seq' for which the 1-argument function 'cond'
    returns a true value."""

    if ___??___: return ___??___

    elif ________: return ________

    else:       return ________
```


Filtering (II)

```
def filter_rlist(cond, seq):
    """The rlist consisting of the subsequence of
    rlist 'seq' for which the 1-argument function 'cond'
    returns a true value."""

    if isempty(seq): return empty_rlist

    elif ___??___: return ________

    else:       return ________
```


Filtering (III)

```
def filter_rlist(cond, seq):
    """The rlist consisting of the subsequence of
    rlist 'seq' for which the 1-argument function 'cond'
    returns a true value."""

    if isempty(seq): return empty_rlist

    elif cond(first(seq)):  return ________

    else:       return ___??___
```


Filtering (IV)

```
def filter_rlist(cond, seq):
    """The rlist consisting of the subsequence of
    rlist 'seq' for which the 1-argument function 'cond'
    returns a true value."""

    if isempty(seq): return empty_rlist

    elif cond(first(seq)):  ___??___

    else: return filter_rlist(cond, rest(seq))
```


Filtering (V)

```
def filter_rlist(cond, seq):
    """The rlist consisting of the subsequence of
    rlist 'seq' for which the 1-argument function 'cond'
    returns a true value."""

    if isempty(seq): return empty_rlist

    elif cond(first(seq)):
        return make_rlist(first(seq),
                          filter_rlist(cond, rest(seq)))
    else: return filter_rlist(cond, rest(seq))
```

- Oops! Not tail-recursive. Iteration is problematic (again).
- In fact, until we get to talking about mutable recursive lists, we won't be able to do it iteratively without creating an extra list along the way.

Python's Sequences

- Rlists are sequences with a particular choice of interface that emphasizes their recursive structure.
- Python has a much different approach to sequences built into its standard data structures, one that emphasizes their *iterative* characteristics.
- There are several different kinds of sequence embodied in the standard types: *tuples*, *lists*, *strings*, *ranges*, *iterators*, and *generators*.
- Python goes to some lengths to provide a uniform interface to all the various sequence types, as well as to its other *collection types*, including *sets* and *dictionaries*.

Sequence Features

- For now, we emphasize computation by *construction* rather than *modification*. The interesting characteristics include:
  - **Explicit Construction:**

```
t = (2, 0, 9, 10, 11)    # Tuple
L = [2, 0, 9, 10, 11]    # List
R = range(2, 13)         # Integers 2-12.
R0 = range(13)           # Integers 0-12.
E = range(2, 13, 2)      # Even integers 2-12.
S = "Hello, world!"      # Strings (sequences of characters)
```

  - **Indexing:**

```
t[2] == L[2] == 9,  R[2] == 4,   E[2] == 6
t[-1] == t[len(t)-1] == 11
S[1] == "e"
```

  - **Slicing:**

```
t[1:4] == (t[1], t[2], t[3]) == (0, 9, 10),
t[2:] == t[2:len(t)] == (9, 10, 11)
t[::2] == t[0:len(t):2] == (2, 9, 11),  t[::-1] == (11, 10, 9, 0, 2)
S[0:5] == "Hello",  S[0:5:2] == "Hlo", S[4::-1] == "olleH"
R[2:5] = range(4, 7), E[1 : 5] = range(4, 12, 2)
```


Sequence Combination and Conversion

- Sequence types can be converted into each other where needed:

```
list( (1, 2, 3) ) == [1, 2, 3],   tuple([1, 2, 3]) == (1, 2, 3)
list(range(2, 10, 2)) == [2, 4, 6, 8]
list("ABCD") = ['A', 'B', 'C', 'D']
```

- One can construct certain sequences (tuples, lists, strings) from smaller ones:

```
A = [ 1, 2, 3, 4 ]
B = [ 7, 8, 9 ]
A + B == [ 1, 2, 3, 4, 7, 8, 9 ]
A[1:3] + B[1:] = [ 1, 2, 3, 8, 9]
(1, 2, 3, 4 ) + (7, 8, 9) = (1, 2, 3, 4, 7, 8, 9)
"Hello," + " " + "world" = "Hello, world"
(1, 2, 3, 4) + 3   ERROR (why?)
```


Sequence Iteration: For Loops

- We can write more compact and clear versions of **while** loops:

```
>>> t = (2, 0, 9, 10, 11)
>>> s = 0
>>> for x in t:
>>>     s += x
>>> print(s)
32
```

- Iteration over numbers is really the same, conceptually:

```
>>> s = 0
>>> for i in range(1, 10):
>>>     s += i
>>> print(s)
45
```


Higher-Order Manipulation of Sequences

- Python 3 defines `map` (just as on rlists), as well as `accumulate` (called `reduce` in the module `functools`), and `filter`, just as we did on rlists.
- So to compute the sum of the even Fibonacci numbers among the first 12 numbers of that sequence, we could proceed like this:

```
First 20 integers:
    0   1   2   3   4   5   6   7   8   9   10   11
Map fib:
    0   1   1   2   3   5   8  13  21  34   55   89
Filter to get even numbers:
    0           2           8          34
Reduce to get sum:
    44
```

- ...or:

```
reduce(add, filter(iseven, map(fib, range(12)))) # or
sum(filter(iseven, map(fib, range(12)))) # Specialized reduction
```

- Why is this important? Sequences are amenable to *parallelization*.

List Comprehensions

- In fact, one doesn't often need `map` and `filter` because Python has a succinct syntax for expressing their application: the list comprehension.
- Full form:

```
[ <expression> for <var> in <sequence expression>
               if <boolean expression> ]
```

- Example: Squares of the prime numbers up to 100.

```
[ x*x for x in range(101) if isprime(x) ]
```

- A different variety is the *generator*, which can be useful in reductions:

```
sum( ( x*x for x in range(101) if isprime(x) ) )
```

...because it does not actually construct the list. More on generators later.

An aside: Sequences in Unix

- Many Unix utilities operate on *streams of characters,* which are sequences.
- With the help of pipes, one can do amazing things. One of my favorites:

```
tr -c -s '[:alpha:]' '[\n*]' < FILE | \
sort | \
uniq -c | \
sort -n -r -k 1,1 | \
sed 20q
```

which prints the 20 most frequently occuring words in *FILE*, with their frequencies, most frequent first.