## Lecture \#5: Higher-Order Functions

## Do You Understand the Machinery? (I)

```
What is printed (0, 1, or error) and why?
def f():
    return 0
def g():
    print(f())
def h():
    def f():
        return 1
    g()
h()
```


## Answer (I)

The program prints 0 . At the point that f is called, we are in the situation shown below:


So we evaluate $f$ in an environment ( fr 2 ) where it is bound to a function that returns 0 .

## Do You Understand the Machinery? (II)

```
What is printed (0, 1, or error) and why?
def f():
    return 0
g = f
def f():
    return 1
print(g())
```


## Answer (II)

The program prints 0 again:


At the time we evaluate $f$ in the assignment to $g$, it has the value indicated by the crossed-out dotted line, so that is the value $g$ gets. The fact that we change $f$ 's value later is irrelevant, just as $x=3 ; y=x ; x$ $=4 ; \operatorname{print}(y)$ prints 3 even though $x$ changes: $y$ doesn't remember where its value came from.

## Do You Understand the Machinery? (III)

```
What is printed (0, 1, or error) and why?
def f():
    return 0
def g():
    print(f())
def f():
    return 1
g()
```


## Answer (III)

This time, the program prints 1 . When $g$ is executed, it evaluates the name ' $f$ '. At the time that happens, $f$ 's value has been changed (by the third def), and that new value is therefore the one the program uses.

## Do You Understand the Machinery? (IV)

What is printed: (1, infinite loop, or error) and why?
def $g(x)$ :
print(x)
def $f(f)$ :
f(1)
$\mathrm{f}(\mathrm{g})$

## Answer (IV)

This prints 1. When we reach $f(1)$ inside $f$, the call expression, and therefore the name $f$, evaluated in the environment $E$, where the value of $f$ is the global function bound to $g$ :

```
def g(x):
        print(x)
def f(f):
        f(1)
f(g)
```



## Do You Understand the Machinery? (V)

```
What is printed: (0, 1, or error) and why?
def f():
    return 0
def g():
    return f()
def h(k):
    def f():
        return 1
    p = k
    return p()
print(h(g))
```


## Answer (V)

This prints 0 . Function values are attached to current environments when they are first created (by lambda or def). Assignments (such as to $p$ ) don't themselves create new values, but only copy old ones, so that when $p$ is evaluated, it is equal to $k$, which is equal to $g$, which is attached to the global environment.

## Observation: Environments Reflect Nesting

- From what we've seen so far:

Linking of environment frames $\Longleftrightarrow$ Nesting of definitions.

- For example, given

```
def f(x):
    def g(x):
        def h(x):
            print(x)
```

The structure of the program tells you that the environment in which $\operatorname{print}(x)$ is evaluated will always be a chain of 4 frames:

- A local frame for $h$ linked to ...
- A local frame for $g$ linked to ...
- A local frame for $f$ linked to ...
- The global frame.
- However, when there are multiple local frames for a particular function lying around, environment diagrams can help sort them out.


## Do You Understand the Machinery? (VI)

What is printed: ( 0,1 , or error) and why?

```
def \(f(p, k)\) :
    def \(g()\) :
        print(k)
    if \(k==0\) :
        \(f(\mathrm{~g}, 1)\)
    else:
        p()
f(None, 0)
```


## Answer (VI)

This prints 0 . There are two local frames for $f$ when $p()$ is called ( $f 1$ and $f 2$ ). The call to $p()$ creates an instantiation of $g$ whose parent is $f 1$.


```
def \(f(p, k):\)
        def \(g()\) :
        print(k)
    if \(k==0\) :
        \(f(\mathrm{~g}, 1)\)
    else:
f(None, 0 )
```


f3: $g[\uparrow f 1]$
Frame for $p()$

## Decorators: Pythonic Use of Higher-Order Functions

- The syntax

```
@expr
def func(expr):
    body
```

is equivalent to ("syntactic sugar for")
def func (expr):
body
func $=($ expr $)$ (func)

- For example, our ucb module defines decorator trace. After

```
from ucb import trace
```

@trace
def mysum (x, y):
return $\mathrm{x}+\mathrm{y}$
mysum will print its arguments and return value each time it is called.

- Usually, expr is a simple name, but it can be any expression that evaluates to a function that takes and returns a function.


## Implement trace

```
def trace(func):
    """A decorator that accepts the same arguments
    and returns the same value as FUNC, but also
    prints the arguments and return value."""
    def afunc(arg):
        print("Call", func.__name__, "with", arg)
        v = func(arg)
        print(func.__name__, "returns", v)
        return v
    return afunc
```


## Implement trace (Fancier Version)

- At the moment, trace handles only one-argument functions.
- To handle more general ones, we use two Python features:

```
def trace(func):
    """A decorator that accepts the same arguments
    and returns the same value as FUNC, but also
    prints the arguments and return value."""
    def afunc(*args): # args is now a list of actual parameters
        print("Call", func.__name_-, "with", args)
        v = func(*args)
        # Line above is like v = func(args[0], args[1], ...)
        print(func.__name_-, "returns", v)
        return v
    return afunc
```


## Design a Decorator

- I'd like a decorator that will check that the output of a function obeys some predicate:
@check_result(lambda x: x < 1000)
def compute(x):
return whatever \# value of whatever must be < 1000 .
- How would you define check_result?
- It must return a function that
- Takes a function, say func, as input
- Returns a function that takes the same arguments as func and returns the same value as func if that value satisfies PRED, but complains otherwise.


## A Decorator That Checks Results

```
@check_result(lambda x: x < 1000)
def compute(x):
    return whatever # value of whatever must be < 1000.
```

- We require that check_result (lambda x: x < 1000) (compute) returns a function that returns the same values as compute, but checks that they are less than 1000 first.
- Let's restrict ourselves to decorating 1-argument functions (like compute).
- The check_result function evidently takes a boolean function (predicate) as its argument:
def check_result(checker):
- And then returns another function that takes a function as its argument and returns a new one:

```
def checked_func(func):
    ?
return checked_func
```


## Checking Decorator (continued)

- And this returned function must return still another function that calls the decorated function (such as compute) and then checks it:

```
def check_result(checker):
    def checked_func(func):
        def call_and_check(x):
            ?
        return call_and_check
    return checked_func
```


## Checking Decorator (completed)

- Final result:

```
def check_result(checker):
    def checked_func(func):
        def call_and_check(x):
            result = func(x)
            if checker(result):
            return result
            else:
                    raise ValueError("bad result") # indicate an error
        return call_and_check
    return checked_func
```


## Higher-Order Functions at Work in Project \#1

This project uses functions to represent aspects of playing a game:

- Strategy: Integer $\times$ Integer $\rightarrow$ Plan (your score, opponent score) $\mapsto$ how to play
- Dice: $\rightarrow$ Integer
() $\mapsto$ random roll of die

