Announcements From Others

CodeBase

"CodeBase is a student-led consultancy that works with local startups to build applications, future product iterations, and develop algorithms. This semester, we're working with three startups to create a cross-platform mobile application, develop an Artificial Intelligence Chatbot, and build data integrations for internet-connected devices like Amazon Alexa. You can find out more about us at codebase.berkeley.edu."

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Official Announcements

- Test #1 is Friday, 17 February, 7–9PM. Rooms to be announced.

Lecture #4: Control (contd.) and Higher-Order Functions

Indefinite Repetition

- With conditionals and function calls, we can conduct computations of any length.
- For example, to sum the squares of all numbers from 1 to N (a parameter):

```
def sum_squares(N):
    """The sum of K**2 for K from 1 to N (inclusive)."""
    if N < 1:
        return 0
    else:
        return N**2 + sum_squares(N - 1)
```

- This will repeatedly call `sum_squares` with decreasing values (down to 1), adding in squares: Execute here

```
sum_squares(3) => 3**2 + sum_squares(2)
               => 3**2 + 2**2 + sum_squares(1)
               => 3**2 + 2**2 + 1**2 + sum_squares(0)
               => 3**2 + 2**2 + 1**2 + 0 => 14
```

Explicit Repetition

- But in the Python, C, Java, and Fortran communities, it is more usual to be explicit about the repetition.
- The simplest form is **while**:

  ```
  while Condition:
      Statements
  ```

  means "If condition evaluates to a true value, execute statements and repeat the entire process. Otherwise, do nothing."

- The effect is (nearly) identical to

  ```
  def loop():
      if Condition:
          Statements
          loop()

  loop()   # Start things off
  ```

- ...except that (for most Python implementations) the latter eventually runs out of memory; and we'll have to do something about assignments to variables in Statements (more on that later).

Sum_squares Iteratively?

- Our original sum_squares was

```
def sum_squares(N):
    """The sum of K**2 for K from 1 to N (inclusive)."""
    if N < 1:
        return 0
    else:
        return N**2 + sum_squares(N - 1)
```

- How do we do the same thing with a **while** loop?

```
def sum_squares(N):
    """The sum of K**2 for K from 1 to N (inclusive)."""
```

Sum_squares Iteratively (II)

```python
def sum_squares(N):
    """The sum of K**2 for K from 1 to N (inclusive)."""
    result = 0
    k = 1
    while k <= N:
        result += k**2
        k += 1
    return result
```

Execute this

Another Way

- Alternatively, I can make this a little shorter by adding the other way:

```
def sum_squares(N):
    """The sum of K**2 for K from 1 to N (inclusive)."""
    result = 0
    while N >= 1:
        result += N**2    # Or result = result + N**2
        N -= 1            # Or N = N-1
    return result
```

Execute here

Functions As Templates

- If we think of a function body as a template for a computation, parameters are "blanks" in that template.
- For example:

```
def sum_squares(N):
    k, sum = 0, 0
    while k <= N:
        sum, k = sum+k**2, k+1
    return sum
```

is a template for an infinite set of computations that add squares of numbers up to 0, 1, 2, 3, . . . , in place of the N.

Functions on Functions

- Likewise, function parameters allow us to have templates with slots for *computations*:

```
def summation(N, f):
    k, sum = 1, 0
    while k <= N:
        sum, k = sum+f(k), k+1
    return sum
```

- Generalizes sum_squares. We can write `sum_squares(5)` as:

```
def square(x):
    return x*x
summation(5, square)
```

- or (if we don't really need a "square" function elsewhere), we can create the function argument anonymously on the fly:

```
summation(5, lambda x: x*x)
```

Lambda

- In Python, **lambda** is just an abbreviation.
- Writing lambda *PARAMS*: *EXPRESSION* is the same as writing NAME, where NAME is a name that appears nowhere else in the program and is defined by

```
def NAME(PARAMS):
    return EXPRESSION
```

  evaluated in the same environment in which the original **lambda** was.

- Now we can write any number of summations succinctly:

```
summation(10, lambda x: x**3)       # Sum of cubes
summation(10, lambda x: 1 / x)      # Harmonic series
summation(10, lambda k: x**(k-1) / factorial(k-1))
                                    # Approximate e**x
```

Functions that Produce Functions

- Functions are *first-class values*, meaning that we can assign them to variables, pass them to functions, and return them from functions.
- Example: let's generalize the class of functions that—like

```
def h(x):  return sin(x) + cos(x)
```

—add the results of applying two functions to the same argument:

```
>>> def add_func(f, g):
...     """Return function that returns F(x)+G(x) for argument x."""
...     def adder(x):               #
...         return f(x) + g(x)      # or return lambda x: f(x) + g(x)
...     return adder                #

>>> from math import sin, cos, pi
>>> h = add_func(sin, cos)
>>> sin(pi/4) + cos(pi/4)
1.414213562373095
>>> h(pi / 4)
1.414213562373095
```

Generalize!

- Let's make a general function-combining function (that goes beyond addition):

```
>>> def combine_funcs(op):
...     """combine_funcs(OP)(f, g)(x) = OP(f(x), g(x))."""
...     def combined(f, g):
...         def val(x):
...             return op(f(x), g(x))
...         return val
...     return combined
```

- Now add_func itself can be constructed by a call to combine_funcs:

```
>>> from operator import add
>>> add_func =
>>> from math import sin, cos, pi
>>> h = add_func(sin, cos)
>>> h(pi / 4)
1.414213562373095
```

- What do the environments look like here? Think about it and try it out.

Generalize!

- Let's make a general function-combining function (that goes beyond addition):

```
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...     """combine_funcs(OP)(f, g)(x) = OP(f(x), g(x))."""
...     def combined(f, g):
...         def val(x):
...             return op(f(x), g(x))
...         return val
...     return combined
```

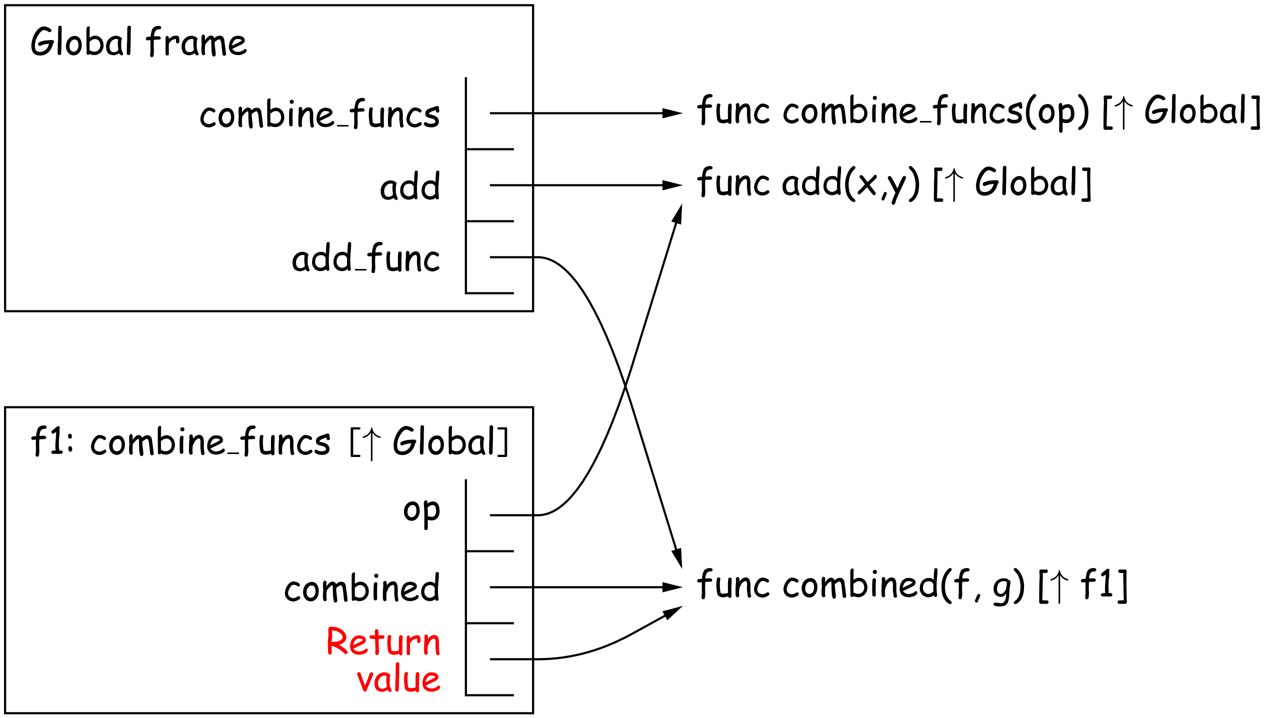
- Now add_func itself can be constructed by a call to combine_funcs:

```
>>> from operator import add
>>> add_func = combine_funcs(add)
>>> from math import sin, cos, pi
>>> h = add_func(sin, cos)
>>> h(pi / 4)
1.414213562373095
```

- What do the environments look like here? Think about it and try it out.

The Environment Picture (I)

```
def combine_funcs(op):
  def combined(f, g):
    def val(x):
      return op(f(x),
g(x))
    return val
  return combined
add_func =
combine_funcs(add)
```



Global frame

combine_funcs

add

add_func

f1: combine_funcs [↑ Global]

op

combined

Return value

func combine_funcs(op) [↑ Global]

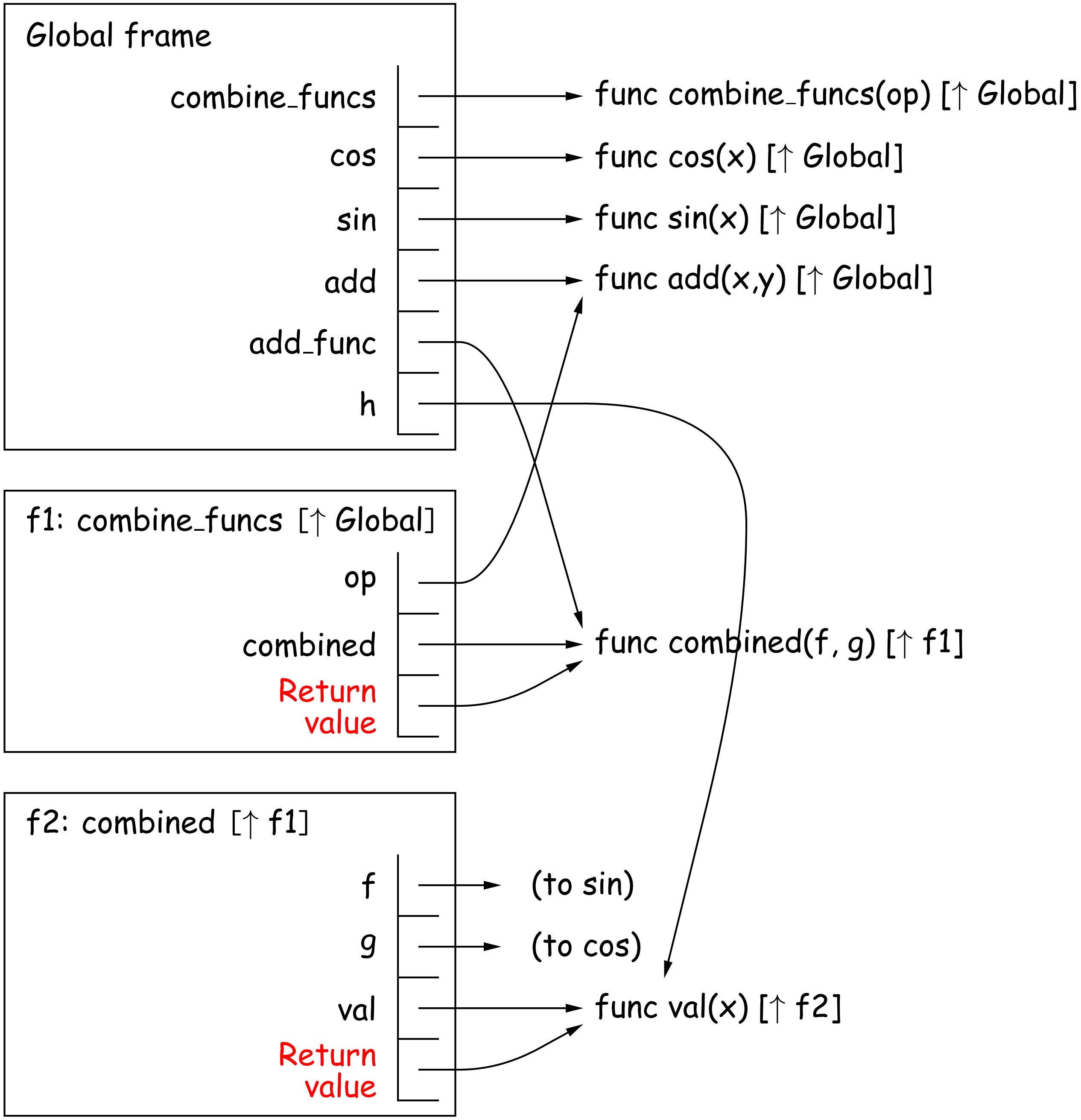
func add(x,y) [↑ Global]

func combined(f, g) [↑ f1]

Legend: ↑ is short for “parent=”.

The Environment Picture (II)

```
def combine_funcs(op):
  def combined(f, g):
    def val(x):
      return op(f(x),
g(x))
    return val
  return combined
add_func =
combine_funcs(add)
h = add_func(sin, cos)
```



Global frame

combine_funcs → func combine_funcs(op) [↑ Global]

cos → func cos(x) [↑ Global]

sin → func sin(x) [↑ Global]

add → func add(x,y) [↑ Global]

add_func

h

f1: combine_funcs [↑ Global]

op

combined → func combined(f, g) [↑ f1]

Return value

f2: combined [↑ f1]

f → (to sin)

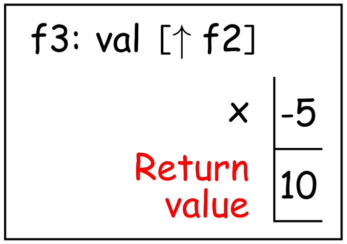
g → (to cos)

val → func val(x) [↑ f2]

Return value

The Environment Picture (III)

```
def combine_funcs(op):
  def combined(f, g):
    def val(x):
      return op(f(x), g(x))
    return val
  return combined
add_func =
combine_funcs(add)
h = add_func(sin, cos)
h(-5)
```



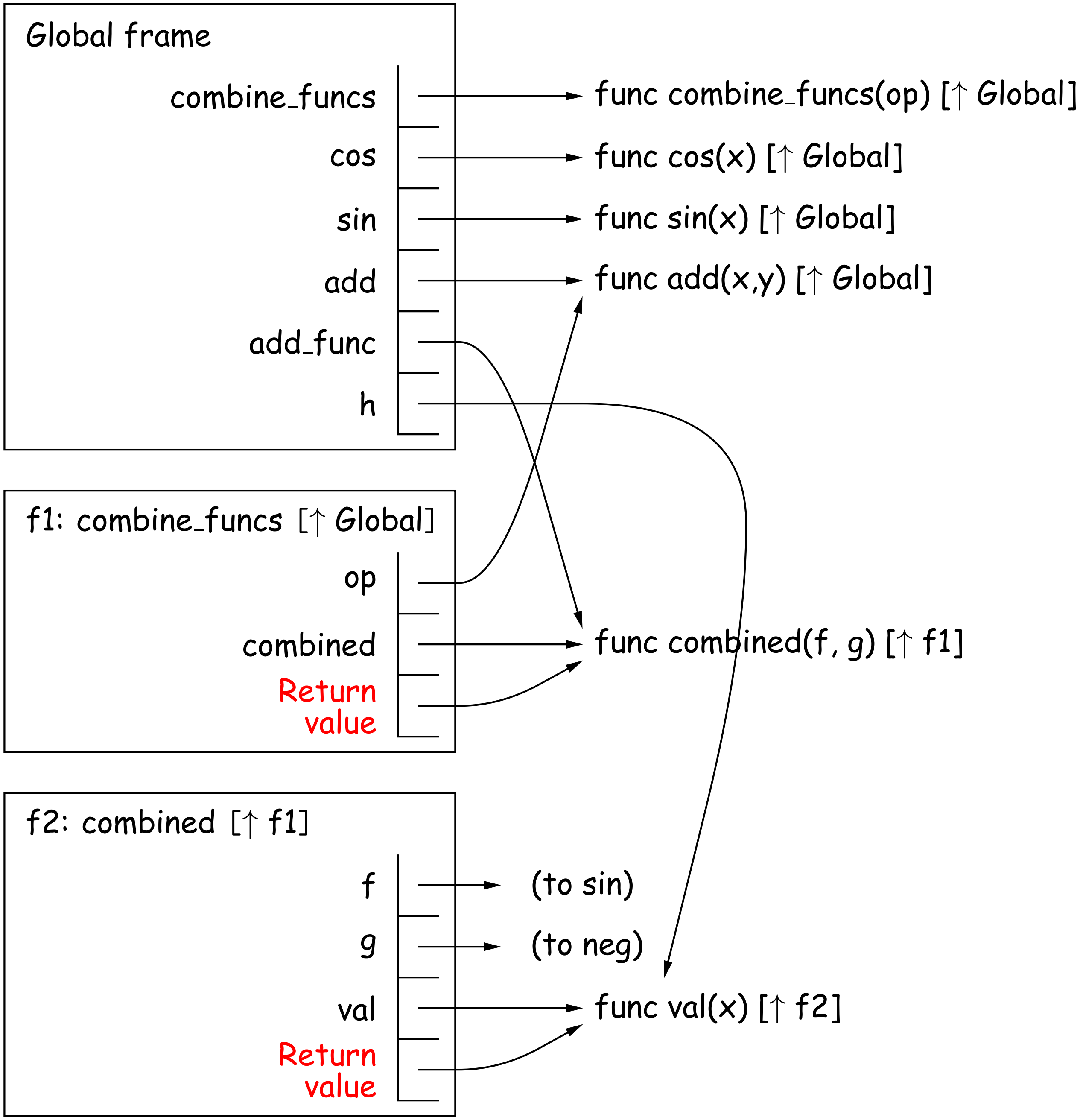
f3: val [↑ f2]

x: -5

Return value: 10

+ local frames for calls to

- add (value of op),
- sin (value of f), and
- cos (value of g)



Global frame

combine_funcs → func combine_funcs(op) [↑ Global]

cos → func cos(x) [↑ Global]

sin → func sin(x) [↑ Global]

add → func add(x,y) [↑ Global]

add_func

h

f1: combine_funcs [↑ Global]

op

combined → func combined(f, g) [↑ f1]

Return value

f2: combined [↑ f1]

f → (to sin)

g → (to neg)

val → func val(x) [↑ f2]

Return value

A Fancy Example

- What does Python print, and why?

```
>>> def chain(n):
...     return lambda which: n if which else chain(n + 1)
>>> g1 = chain(1)
>>> g1(True)

________
>>> g2 = g1(False)
>>> g2

________
>>> g2(True)

________
>>> g2(False)(True)

________
```

A Fancy Example

- What does Python print, and why?

```
>>> def chain(n):
...     return lambda which: n if which else chain(n + 1)
>>> g1 = chain(1)
>>> g1(True)
    1
>>> g2 = g1(False)
>>> g2

>>> g2(True)

>>> g2(False)(True)

```

A Fancy Example

- What does Python print, and why?

```
>>> def chain(n):
...     return lambda which: n if which else chain(n + 1)
>>> g1 = chain(1)
>>> g1(True)
____1____
>>> g2 = g1(False)
>>> g2
<function chain...>
>>> g2(True)

_________
>>> g2(False)(True)

_________
```

A Fancy Example

- What does Python print, and why?

```
>>> def chain(n):
...     return lambda which: n if which else chain(n + 1)
>>> g1 = chain(1)
>>> g1(True)
    1
>>> g2 = g1(False)
>>> g2
<function chain...>
>>> g2(True)
    2
>>> g2(False)(True)
________
```

A Fancy Example

- What does Python print, and why?

```
>>> def chain(n):
...     return lambda which: n if which else chain(n + 1)
>>> g1 = chain(1)
>>> g1(True)
    1
>>> g2 = g1(False)
>>> g2
<function chain...>
>>> g2(True)
    2
>>> g2(False)(True)
    3
```