Lecture #2: Functions, Expressions, Environments

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From Last Time

- From last lecture: *Values* are data we want to manipulate and in particular,
- *Functions* are values that perform computations on values.
- *Expressions* denote computations that produce values.
- Today, we'll look at them in some detail at how functions operate on data values and how expressions denote these operations.
- As usual, although our concrete examples all involve Python, the actual concepts apply almost universally to programming languages.

Functions

- For this lecture, we're going to use this notation to show function *values* (which are created by evaluating function *definitions*):

  (We'll simplify this in a bit to make it easier to write.)

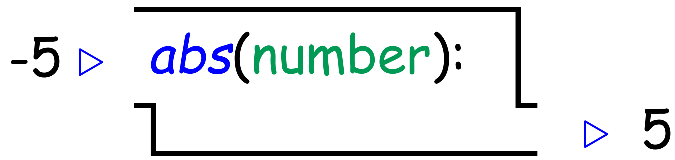
- The green parenthesized lists indicate the number of *parameter values* or *inputs* the functions operate on (this information is also known as a function's *signature*).

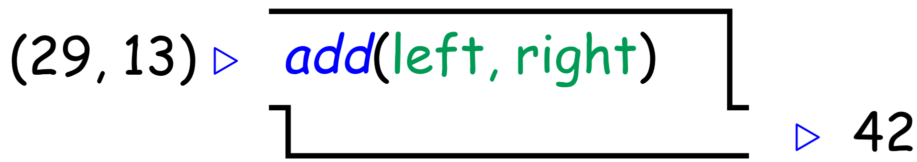
- For our purposes, the blue name is simply a helpful comment to suggest what the function does, and the specific (green) parameter names are likewise just helpful hints.

- (Python actually maintains this *intrinsic name* and the parameter names internally, but this is not a universal feature of programming languages.)

Pure Functions

- The fundamental operation on function values is to *call* or *invoke* them, which means giving them one value for each formal parameter and having them produce the result of their computation on these values:

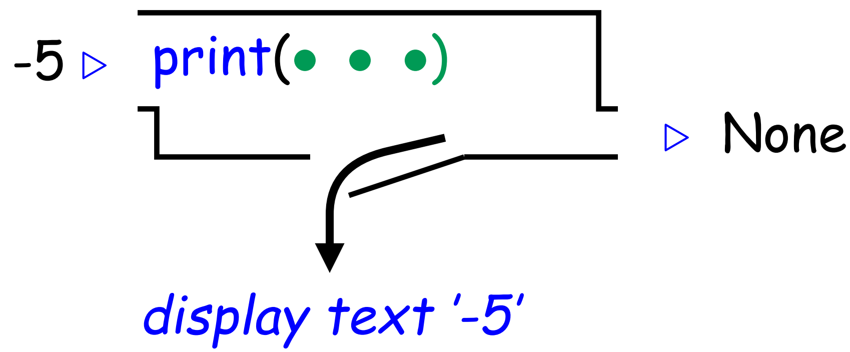




- These two functions are *pure:* their output depends only on their input parameters' values, and they do nothing in response to a call but compute a value.

Impure Functions

- Functions may do additional things when called besides returning a value.
- We call such things *side effects.*
- Example: the built-in print function:



- Displaying text is print's side effect. Its value, in fact, is generally useless (always the null value).

Other Kinds of Impurity

- Most side-effects involve changing the value of some variable.
- Example: the function random.randint:

```
>>> random.randint(0, 100)   # Random number in 0--100.
13
>>> random.randint(0, 100)
55                           # Something must have changed!
```

Call Expressions

- A call expression denotes the operation of calling a function.
- Consider add(2, 3):

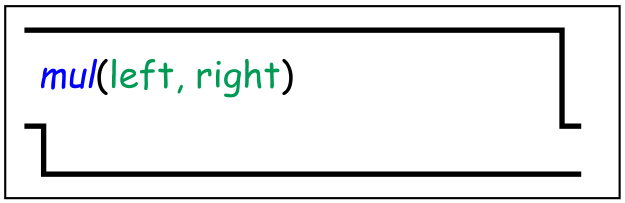
$$\underbrace{\text{add}}_{\textit{Operator}}(\underbrace{2}_{\textit{Operand 0}},\underbrace{3}_{\textit{Operand 1}})$$

- The operator and the operands are all themselves expressions (recursion again).
- To evaluate this call expression:
  - Evaluate the operator (let's call the value C);
  - Evaluate the operands in the order they appear (let's call the values P_0 and P_1)
  - Call C (which must be a function) with parameters P_0 and P_1.
- Together with the definitions for base cases (mostly literal expressions and symbolic names), this describes how to evaluate any call.

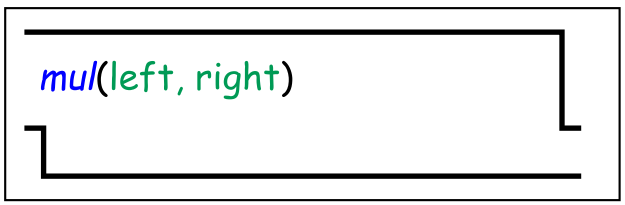
Example: From Expression to Value

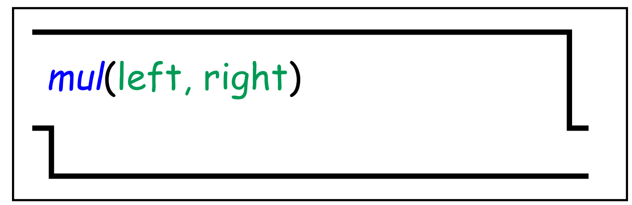
Let's evaluate the expression mul(add(2, mul(0x4, 0x6)), add(0x3, 005)).
In the following sequence, values are shown in boxes.
Everything outside a box is an expression.

- mul(add(2, mul(0x4, 0x6)), add(0x3, 005))

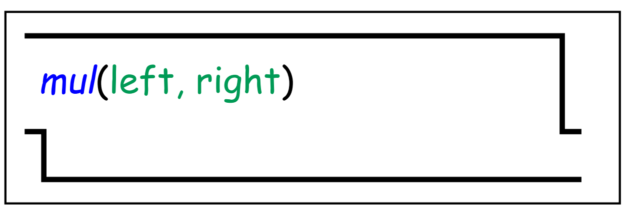


- (add(2, mul(0x4, 0x6)), add(0x3, 005))





- ((2 , (4 6)), add(0x3, 005))



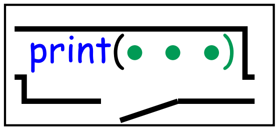
- ((2 , 24), add(0x3,

005))

- mul(left, right) (26 , add(0x3, 005))
- mul(left, right) (26 , add(left, right) (3 , 5))
- mul(left, right) (26 , 8)
- 208 .

Example: Print

What about an expression with side effects?



1. print(print(1), print(2))
2. ((1), print(2))
3. (None , print(2))
   and print '1'.
4. (None , (2))
5. (None , None))
   and print '2'.
6. None
   and print 'None None'.

Names

- Evaluating expressions that are literals is easy: the literal's text gives all the information needed.
- But how did I evaluate names like add, mul, or print?
- Deduction: there must be another source of information.
- We'll first try a simple approach: *substitution* of values for names.
- This won't cover all the cases, however, and so we'll introduce the concept of an *environment*.

Substitution

- Let's try to explain the effect of

```
x = 3
y = x * 2
z = y ** x
```

  by treating each assignment (=) as a *definition*.

- Thus, we get

```
x = 3          x = 3          x = 3          x = 3
y = x * 2      y = 3 * 2      y = 6          y = 6
z = y ** x     z = y ** 3     z = 6 ** 3     z = 216
```

- That is, we *replace names by their definitions (values).*

Substitution and Functions

- Now consider a simple function definition:

```
def compute(x, y):
    return (x * y) ** x
print(compute(3, 2))
```

- A **def** statement is sort of like an assignment, but specialized to functional values.
- The **def** statement above defines compute to be "the function of x and y that returns $(xy)^x$."
- Here, I'll use a common notation for that (due to Church):

$$\lambda\ x, y : (xy)^x.$$

- So after substitution for compute, we have

print($(\lambda\ x, y : (xy)^z)$ (3, 2))

- Now what?

Substitution and Formal Parameters

- A function call such as

  $(\lambda\ x, y : (xy)^z)$ (3, 2)

  from last slide is like a *simultaneous assignment* to or substitution for x and y.

- So we replace the whole expression with

  $(3 \cdot 2)^3$

  and (eventually), just 216.

Getting Fancy

- What about this?

```
def incr(n):
    def f(x):
        return n + x
    return f

print(incr(5)(6))
```

- The incr function returns a function. The argument to **print** then calls this function on 6.
- What happens?

Answer

- First, deal with incr:

```
def incr(n):
    def f(x):
        return n + x
    return f
```

```
print(incr(5)(6))        print((λ n: return λ x: n + x)(5)(6))
```

- The 5 now gets substituted for n:

```
print((λ x: 5 + x)(6)
```

- And 6 for x:

```
print(5 + 6)
```

- Finally giving

```
print(11)
```

Trouble

- Alas, this relatively simple (if tedious) approach doesn't work.
- Example:

```
x = 4
x = 8
print(x)
```

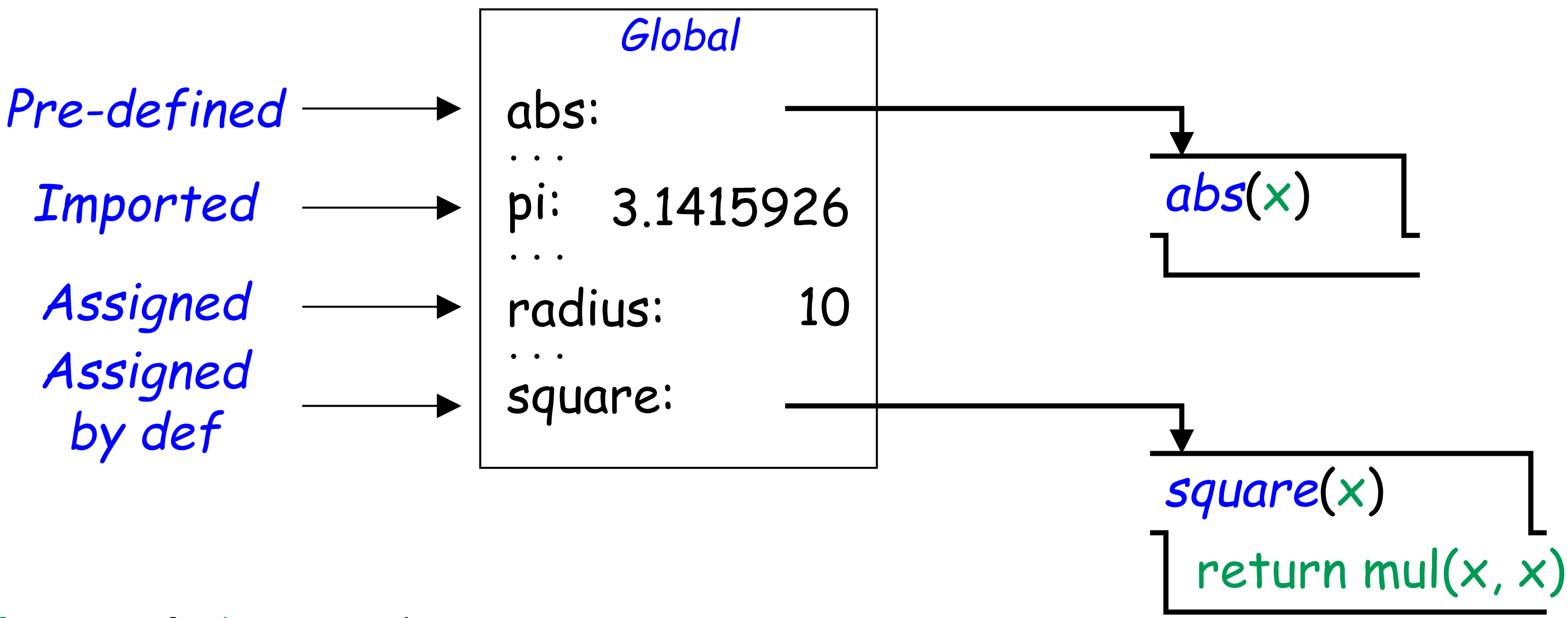
- If we just substitute for the first x as before:

```
x = 4
x = 8    # or even worse: 4 = 8
print(4)
```

- ...we get a wrong result (4 instead of 8).
- After one substitution, x isn't around any more to substitute for.
- We need something stronger.

Environments

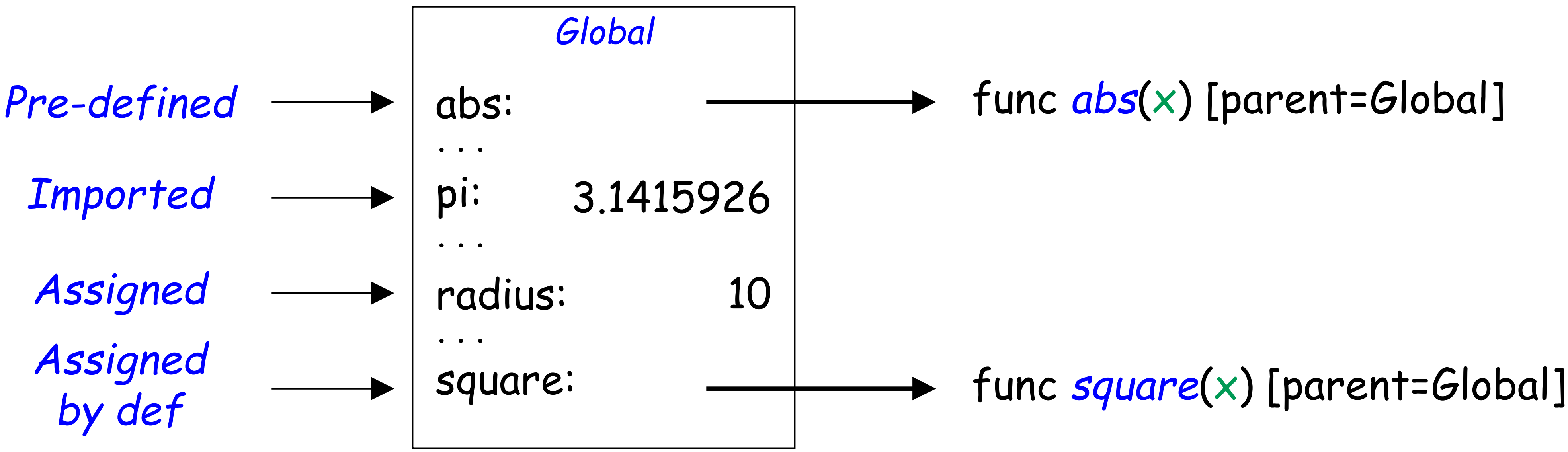
- An environment is a mapping from names to values.
- We say that a name is *bound to* a value in this environment.
- In its simplest form, it consists of a single *global environment frame*:




```
from math import pi
radius = 10
def square(x): return x**2
```

Slight Change of Notation

- You'll be using the Python Tutor from time to time, which uses a somewhat different notation for function values. Might as well get used to it (we'll explain the "parent=" stuff in a later lecture):




```
from math import pi
radius = 10
def square(x): return x**2
```

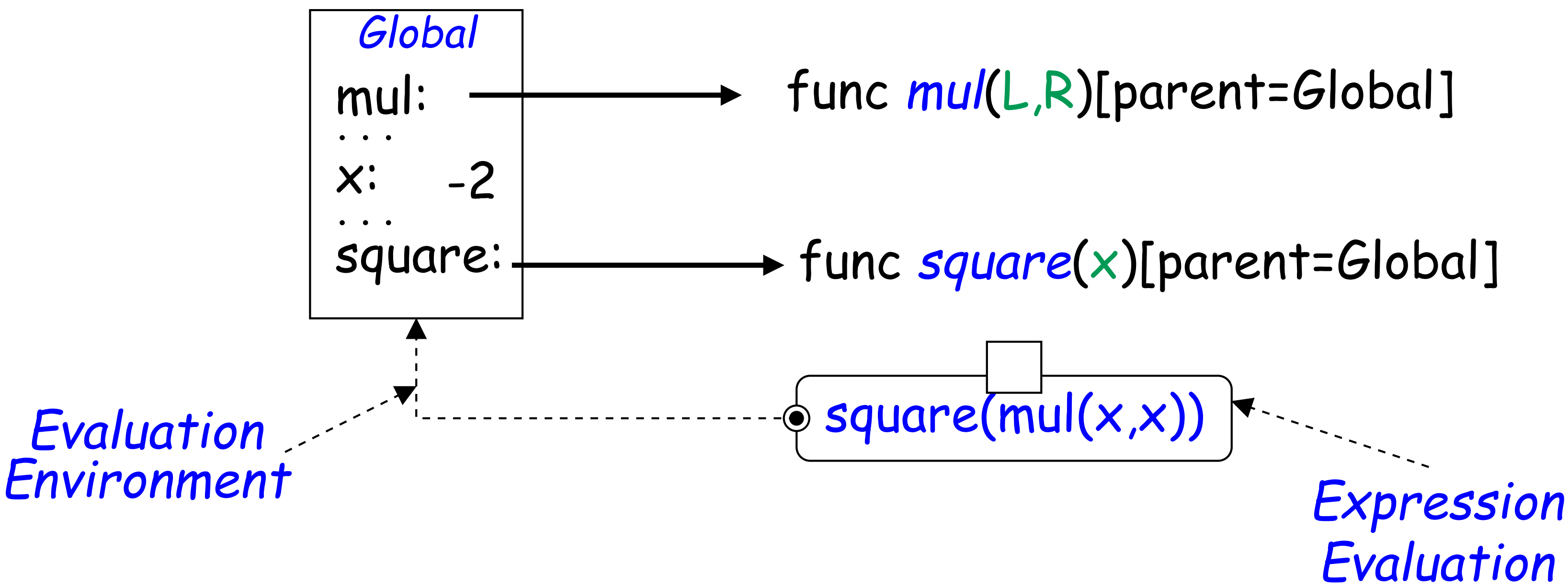
Environments and Evaluation

- Every expression is evaluated in an environment, which supplies the meanings of any names in it.
- Evaluating an expression typically involves first evaluating its subexpressions (the operators and operands of calls, the operands of conventional expressions such as `x*(y+z)`, ...).
- These subexpressions are evaluated in the same environment as the expression that contains them.
- Once their subexpressions (operator + operands) are evaluated, calls to user-defined functions must evaluate the expressions and statements from the definition of those functions.

Evaluating User-Defined Function Calls

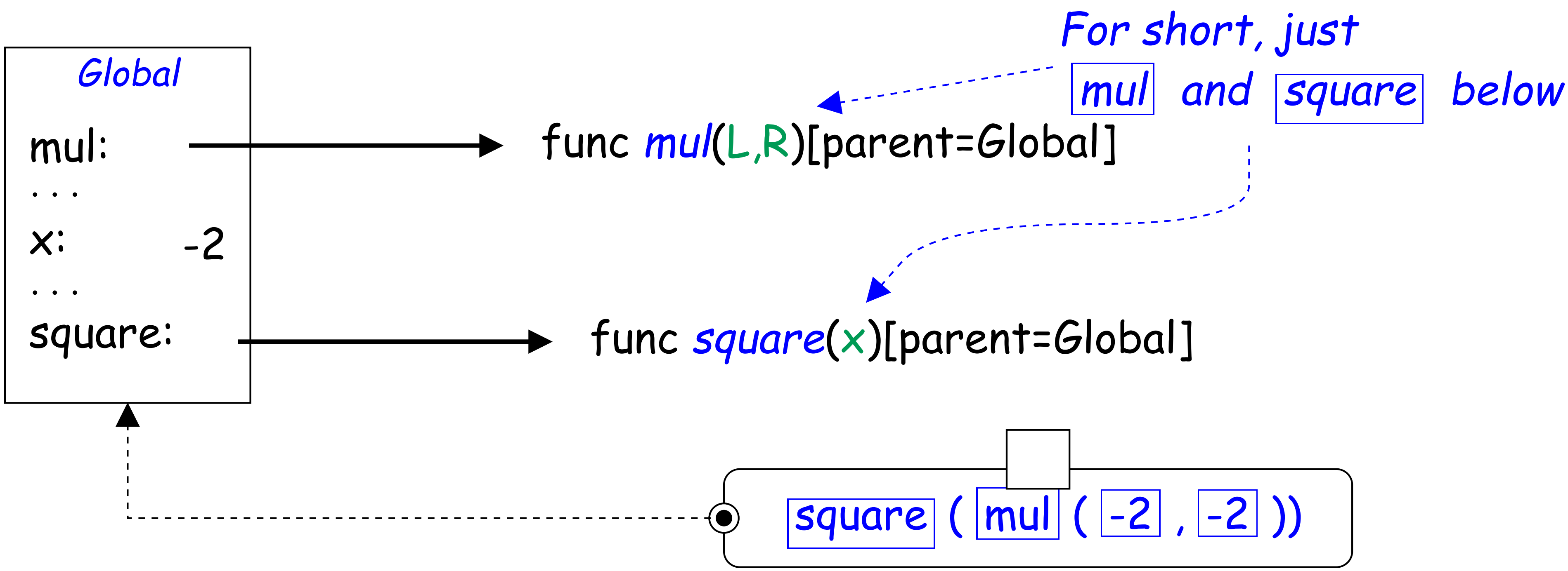
- Consider the expression square(mul(x, x)) after executing

```
from operator import mul
def square(x):
    return mul(x,x)
x = -2
```



Evaluating User-Defined Function Calls (II)

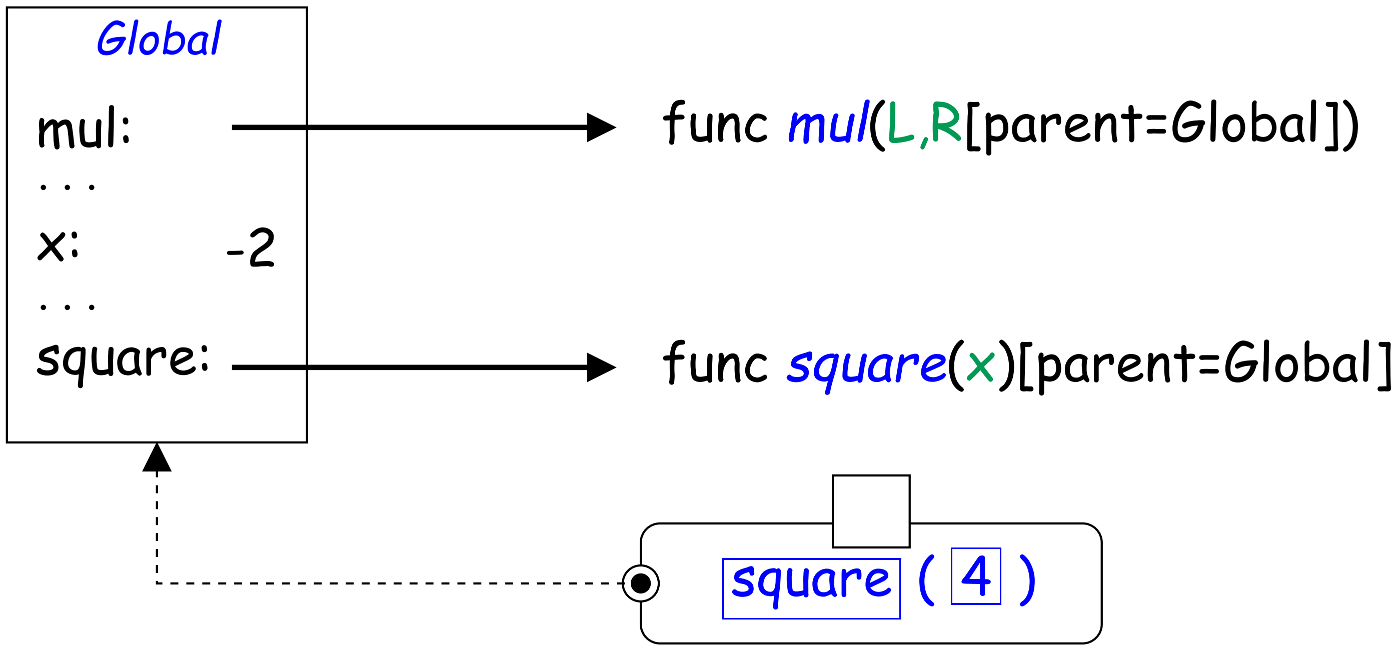
- First evaluate the subexpressions of square(mul(x, x)) in the global environment:



- Evaluating subexpressions x, mul, and square take values from the expression's environment.

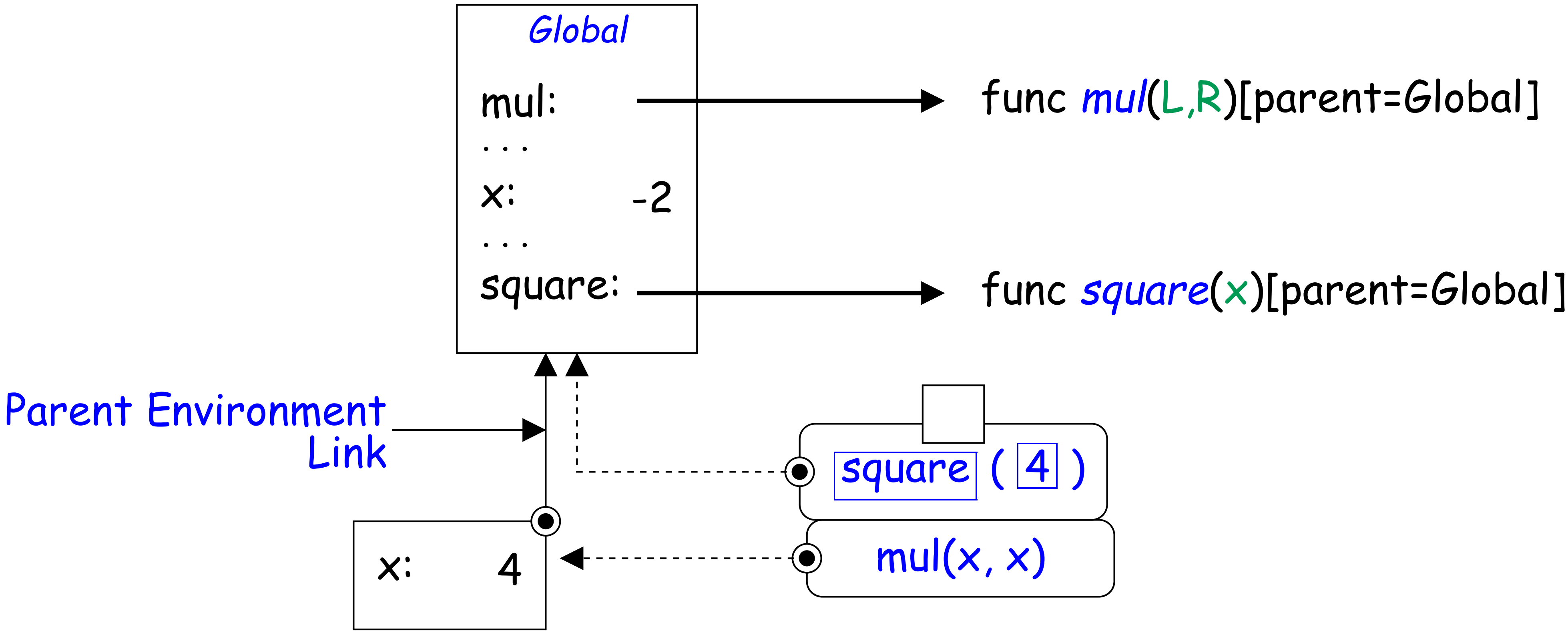
Evaluating User-Defined Functions Calls (III)

- Then perform the primitive multiply function:



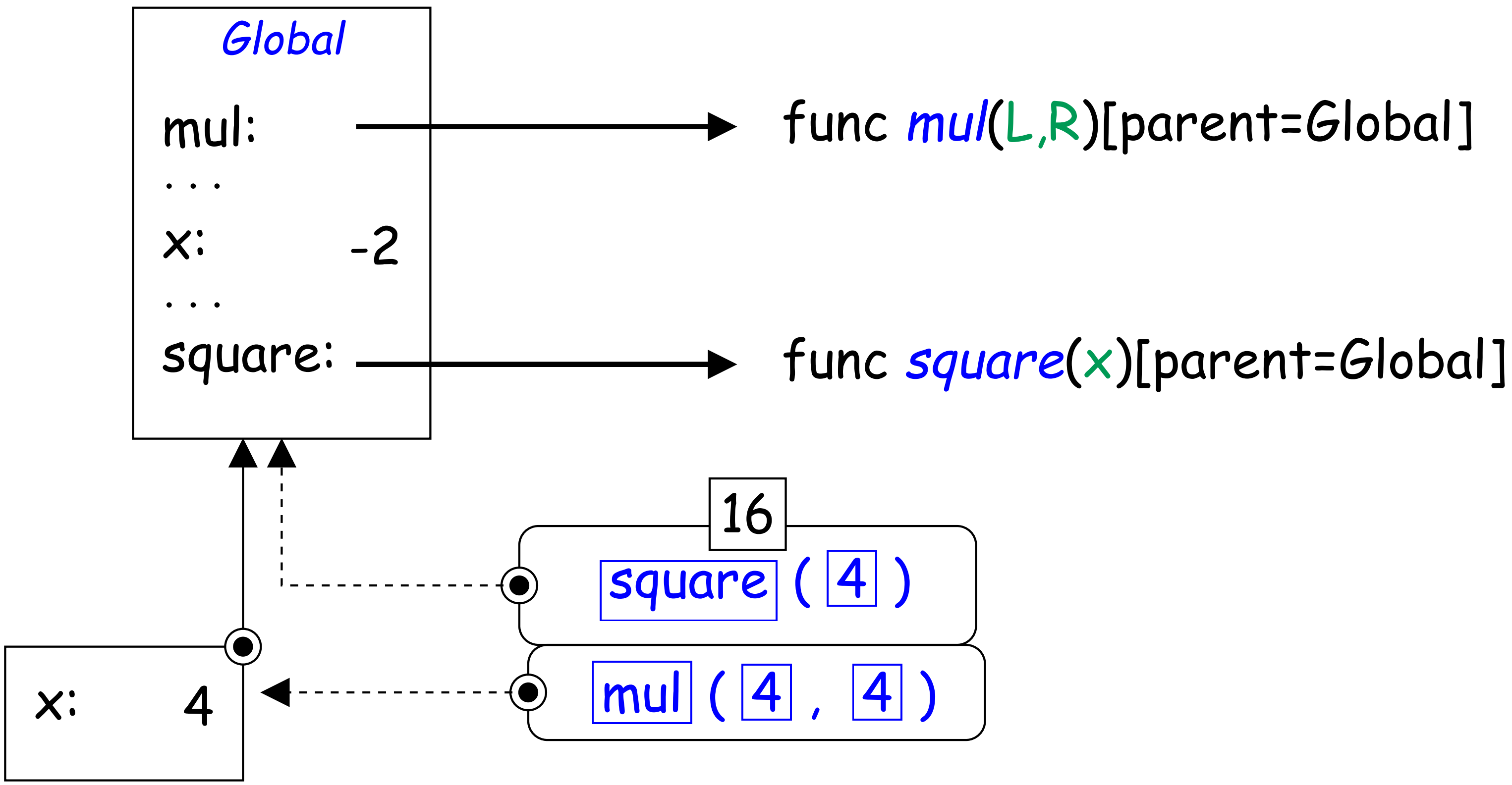
Evaluating User-Defined Functions Calls (IV)

- To explain parameter to user-defined square function, extend environment with a *local environment frame*, attached to the frame in which square was defined (the global one in this case), and giving x the operand value.
- Now replace original call with evaluating body of square in the new local environment.



Evaluating User-Defined Functions Calls (V)

- When we evaluate mul(x, x) in this new environment, we get the same value as before for mul, but the local value for x.
- When evaluating an identifier in a chain of environments, follow the parent environment links to the first frame containing its definition.



So How Does This Help?

- The original problem that led to this whole environment diagram thing was how to deal with:

```
x = 4
x = 8
print(x)
```

- Now it's easy. Each time we assign to *x*, we create a new binding for it in the current evaluation frame (replacing the old one, if any).
- We get the new (last assigned) value when we look up *x* in the modified environment.